2.2 SEISMIC RESOLUTION OF ZERO-PHASE WAVELETS

2.3 DESIGNING OPTIMUM ZERO-PHASE WAVELETS

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QUIZ

1. FIND THE THICKNESS OF A CARBONATE BED ENCASED IN SHALE.

GIVEN:

- SEISMIC SECTION WHITENED BETWEEN 5 Hz & 60 Hz
- 11 ms MEASURED TWO-WAY INTERVAL TIME
- 18,000 ft/sec INTERVAL VELOCITY OF BED

FIND:

- THE THICKNESS OF THE BED

☐ 99 ft
☐ 49.5 ft
☐ 18 ft
☐ ANY ONE OF THE ABOVE
☐ NONE OF THE ABOVE

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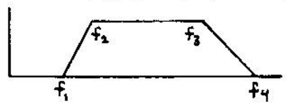
2. DEFINE THE ORMSBY WAVELET (f_1, f_2, f_3, f_4)



HAVING THE SAME RESOLVING POWER AS A 65 Hz RICKER WAVELET OUTPUT FROM PROGRAM GENF.

	f_1	f_2	f_3	f_4
☐	24	26	99	101
☐	11	13	88	110
☐	4	8	60	121
☐	14	18	31	125

☐ ALL OF THE ABOVE
☐ NONE OF THE ABOVE
☐ ROLL YOUR OWN:

___ ___ ___ ___

SEISMIC RESOLUTION OF ZERO-PHASE WAVELETS

SUMMARY

The subject of seismic resolution in the context of thin bed resolvability has not been adequately covered in the literature. The term "temporal resolution" is suggested to describe this type of resolution and is defined as follows:

> The temporal resolution of a zero-phase wavelet is the time interval between the wavelet's primary lobe inflection points; it is equivalent to the minimum two-way time through a thinning bed as measured on a seismic trace. A wavelet's inflection points may be found by setting the wavelet's second derivative equal to zero.

Different types of zero-phase wavelets may be compared in terms of their temporal resolution; that is to say, their ability to resolve thin beds can be separated from variations in sidelobe tuning effects.

DISCUSSION

The question under consideration is the following: As interval times through a thinning bed become less and less, how accurately do the measured times represent the actual, vertical two-way travel times through the bed.

This question may be further divided into two related questions:

1. How thin can a bed become and still be resolvable? In other words, when is the measured interval time essentially the same as the true interval time?

2. What are the errors between the true interval times and the measured interval times through thick beds?

The use of wavelets that are not zero-phase complicates the analysis of seismic resolution greatly. The use of zero-phase wavelets simplifies resolution because traces containing zero-phase wavelets will have seismic interfaces located in general at the centers of the peaks and troughs of the trace (neglecting tuning effects and noise).

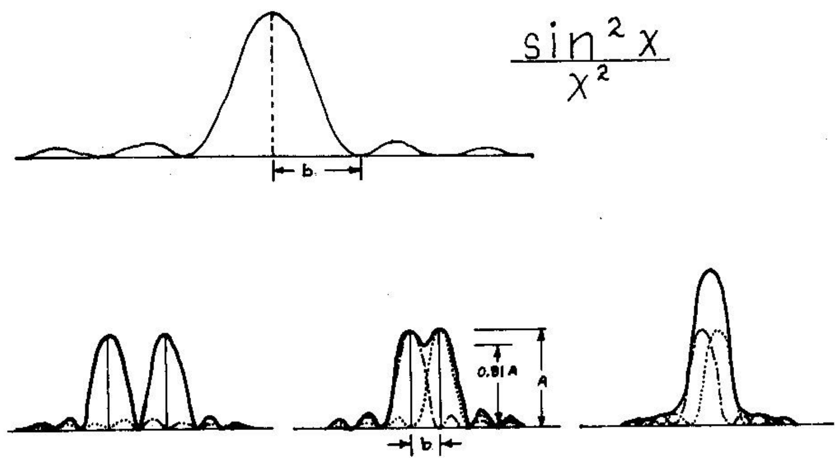
Since past work on the subject of seismic resolution is considered by many investigators to be definitive, our new concepts and results will be examined carefully and compared with those in the literature. We begin our study in the field of optics by examining the Rayleigh criterion of resolution, and consider next a resolution criterion developed by Ricker (1953), and finally the criterion established by Widess in 1957 and published again in Geophysics in 1973. In each case, the theoretical limits of resolution will be related to parameters that can be measured on the wavelet that is convolved with the reflectivity sequence.

Fig. 1. RAYLEIGH'S CRITERION

Although optical diffraction patterns caused by light transmitting through a narrow slit may seem far removed from seismic resolution, the criterion of resolution established has often been cited by many investigators in regard to seismic wavelets. The text book "Fundamentals of Optics" by Jenkins & White (1957) is a good reference.

RAYLEIGH'S CRITERION

(RELATES TO RESOLUTION OF TWO DIFFRACTION PATTERNS)



TWO WAVELETS ARE RESOLVED WHEN THEIR SEPARATION $\geq$ THE PEAK-TO-TROUGH TIME OF THE CONVOLVING WAVELET.

<u>COMMENT:</u> A QUITE ARBITRARY CHOICE BY RAYLEIGH TO KEEP THE MATHEMATICAL RELATIONSHIPS INVOLVED SIMPLE.

WHEN APPLIED TO WAVELETS OTHER THAN $\frac{\sin^2 x}{x^2}$, THE "DIMPLE-TO-PIMPLE" AMPLITUDE RATIOS MAY VARY.

FIG. 1

Fig. 2. RICKER'S CRITERION

A bed, represented by two upright polarity spikes convolved with a wavelet, reaches the limit of resolvability when the bed becomes so thin as to cause a flat spot to appear in place of the two maxima. This occurs at a spike separation interval that can be derived equating to zero the second derivative of the convolving wavelet. This observation was made first by Ricker in his classic paper "Wavelet Contraction, Wavelet Expansion, and the Control of Seismic Regulation" by Norman Ricker (1953) Geophysics, Vol. 18, No. 4, p. 769-792.

Ricker also recognized that as a bed, represented by two spikes of equal amplitudes but opposite polarities, becomes thinner and thinner, the complex waveform produced by convolving the spike pair with a wavelet looks more and more like the time derivative of the convolving wavelet. It was left to Widess to expand on this concept further in his paper: "How Thin Is A Thin Bed" by M. B. Widess (1973) Geophysics, Vol. 38, No. 6, P. 1176-1180.

RICKER'S CRITERION *
(RELATES TO RESOLUTION OF TWO "RICKER" WAVELETS)

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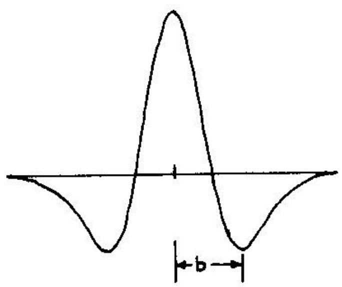
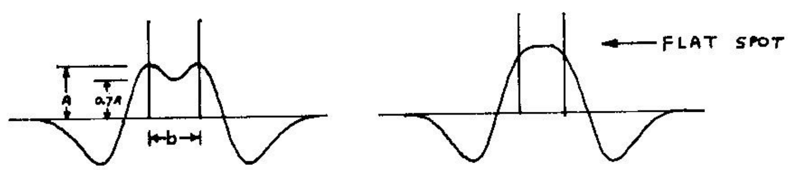


$$Y(f) = \left(\frac{f}{f_1}\right)^2 e^{1-(f/f_1)^2}$$



RAYLEIGH'S CRITERION

RICKER'S CRITERION

$$\frac{d^2(K_t)}{dt^2} = 0$$

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* RICKER, N., WAVELET CONTRACTION, WAVELET EXPANSION, AND THE CONTROL OF SEISMIC RESOLUTION, GEOPHYSICS OCTOBER 1953, VOL. 18, PP. 769-792.

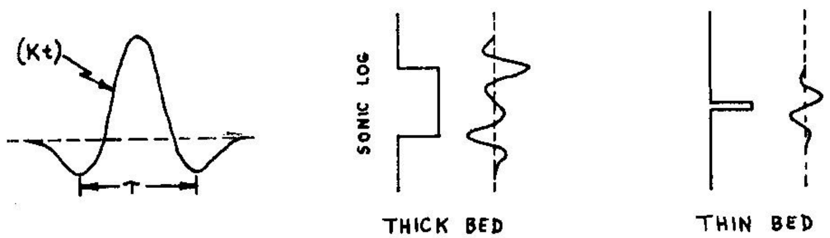
FIG. 2

Fig. 3. THE WIDESS CRITERION

Widess considered a thin bed as one where the complex waveform across it does not differ significantly from the derivative of the convolving wavelet itself. This definition is useful for thin bed "detectability" studies, but causes problems when it comes to thin bed "resolvability" considerations. At the bed thickness Widess first considers a bed to become a "thin" bed, i.e., when the bed thickness is about $\lambda_b/8$; the apparent thickness is actually $\lambda_b/4.6$ which is the peak-to-trough time of the derivative of a Ricker wavelet. J. Farr in his paper: "How High Is High Resolution" (1976) SEG Preprint, states that a bed as thin as $\lambda_b/40$ may be detectable. It should be understood, however, that the apparent thickness remains at $\lambda_b/4.6$.

THE WIDESS CRITERION*

THE COMPLEX WAVEFORM ACROSS A THIN BED APPROACHES THE TIME DERIVATIVE OF THE INCIDENT WAVELET AS THE BED THINS TO ZERO THICKNESS.



T = "PREDOMINANT PERIOD"
$1/T$ = "PREDOMINANT FREQUENCY". NOT TO BE CONFUSED WITH PEAK FREQUENCY.

WIDESS STATES, "A THIN BED IS ONE WHOSE THICKNESS IS LESS THAN ABOUT $\lambda_b/8$ WHERE λ_b IS THE PREDOMINANT WAVELENGTH..."

<u>COMMENT</u>: THE MINIMUM TIME DIRECTLY MEASURABLE THROUGH A "THIN" BED MAY BE CALCULATED FROM $\frac{d^2(Kt)}{dt^2} = 0$

FOR A RICKER WAVELET THEN, A "THIN" BED IS ONE WHICH HAS A THICKNESS LESS THAN

$$\lambda_b / 4.6$$

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* WIDESS, M.B., HOW THIN IS A THIN BED?, GEOPHYSICS, DEC. 1973, VOL. 38.

FIG. 3

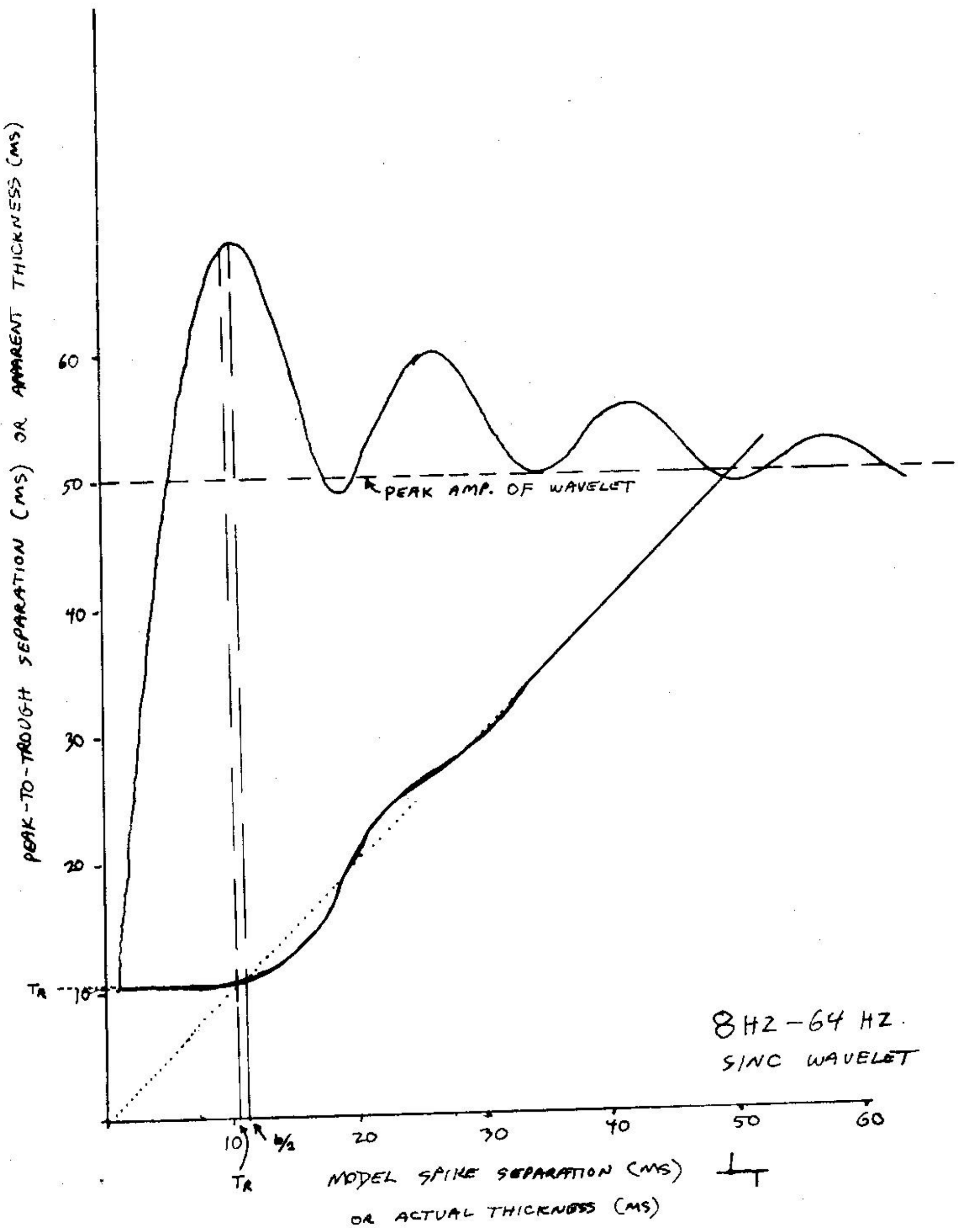
TEMPORAL RESOLUTION

We now propose a definition of seismic resolution, in the context of thin bed resolvability that ties together both Ricker's and Widess' criteria and relates both of them to parameters that can be measured on the incident wavelet itself.

In order to separate the concept of "resolvability" from the related concept of "detectability", the term Temporal Resolution will be used to denote resolvability.

The term Temporal Resolution of a zero-phase wavelet may be defined as the time interval between the wavelet's primary lobe inflection points. This is the minimum two-way time through a thinning bed as measured directly on a seismic trace. A wavelet's inflection points are found by setting equal to zero the second derivative of the wavelet itself. Fig. 4 shows the temporal resolution of a Ricker wavelet in terms of the wavelet's peak frequency (f_1). Peak frequency is not to be confused with the term "predominant frequency" which is defined in the literature as the reciprocal of the wavelet's breadth (T_b).

Let us now discuss temporal resolution in relation to the band-pass sinc wavelet. This wavelet represents the desired output of an amplitude whitening process such as programs DAFD or WELCON.



PEAK-TO-TROUGH SEPARATION (MS) OR APPARENT THICKNESS (MS)
60
50
40
30
20
TR
10
PEAK AMP. OF WAVELET
8 HZ – 64 HZ.
SINC WAVELET
10
TR
b/2
20
30
40
50
60
MODEL SPIKE SEPARATION (MS)
OR ACTUAL THICKNESS (MS)

TEMPORAL RESOLUTION (T_R)

OR

(THIN BED RESOLVING POWER)

THE TEMPORAL RESOLUTION OF A WAVELET MAY BE DEFINED AS THE TIME BETWEEN THE WAVELET'S (PRIMARY LOBE) INFLECTION POINTS.

THIS TIME MAY BE DERIVED FROM THE EQUATION:

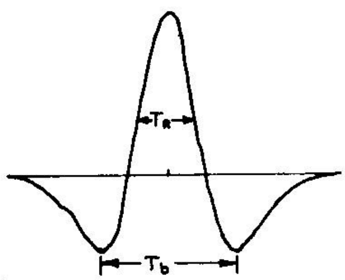
$$\frac{d^2(Kt)}{dt^2} = 0$$

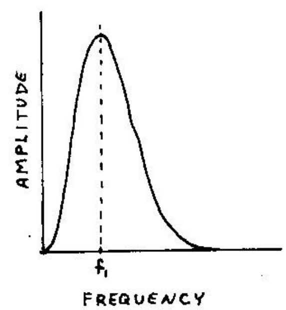
In Fig. 5, the low-pass sinc wavelet is analyzed in terms of its mid-frequency in order to establish a similarity to the analysis of the Ricker wavelet. A low-pass sinc wavelet is not realizable in actual practice because it has frequencies extending to zero Hertz; nevertheless, it is instructive to study this wavelet. Temporal resolution is established in terms of the maximum frequency, and the results used in the discussion of Fig. 6.

TEMPORAL RESOLUTION (T_R)

THE RICKER WAVELET

$$K_t = [1 - 2(\pi f_1 t)^2]\, e^{-(\pi f_1 t)^2}$$





TEMPORAL RESOLUTION $T_R = \frac{1}{3.0 f_1}$

WAVELET BREADTH $T_b = \frac{1}{1.3 f_1}$

PEAK-TO-TROUGH $T_b/2 = \frac{1}{2.6 f_1}$

RELATIONSHIP OF (T_b) TO (T_R):

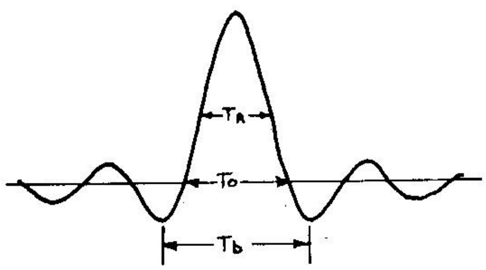
$$T_R = 0.43\, T_b = 0.86\, T_b/2$$

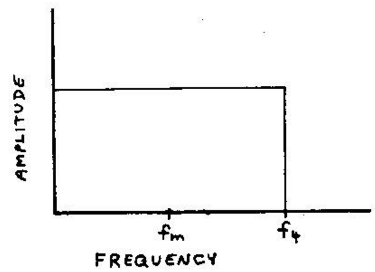
FIG. 4

TEMPORAL RESOLUTION (T_R)

THE LOW-PASS SINC WAVELET

$$K_t = \frac{2f_4 \sin(2\pi f_4 t)}{(2\pi f_4 t)}$$





		[MID FREQUENCY]		[MAX. FREQUENCY]
TEMPORAL RESOLUTION	$T_R =$	$\frac{1}{3.0 f_m}$	$=$	$\frac{1}{1.5 f_4}$
WAVELET BREADTH	$T_b =$	$\frac{1}{1.4 f_m}$	$=$	$\frac{1}{0.7 f_4}$
PEAK-TO-TROUGH	$T_b/2 =$	$\frac{1}{2.8 f_m}$	$=$	$\frac{1}{1.4 f_4}$
1st ZERO CROSSINGS	$T_o =$	$\frac{1}{2 f_m}$	$=$	$\frac{1}{f_4}$

RELATIONSHIP OF (T_b) TO (T_R):

$$T_R = 0.47\, T_b = 0.93\, T_b/2$$

FIG. 5

Fig. 6 defines the band-pass sinc wavelet as the difference between two sinc functions. It is shown in Figs. 7 and 8 that the f_1 sinc function has negligible effect on the temporal resolution of the wavelet for band-pass ratios of 2-octaves and greater. The ability to relate temporal resolution to the highest, and only the highest, frequency of a wavelet leads to some very useful and quite accurate approximations as indicated in Fig. 6.

Let us now consider the effect on temporal resolution when the amplitude of the second spike of a set of alternate polarity spike pairs is varied. Fig. 9 shows that temporal resolution decreases slightly and in fact approaches the peak-to-trough time of the convolving wavelet. This is due to the constructive interference of the main trough of the positive spike wavelet on the center lobe of the negative spike wavelet. Note also the enlargement of the "pseudo-thinning" pocket just prior to temporal resolution. Application of these temporal resolution concepts is discussed in the attached report on "Designing Optimum Zero-Phase Wavelets".

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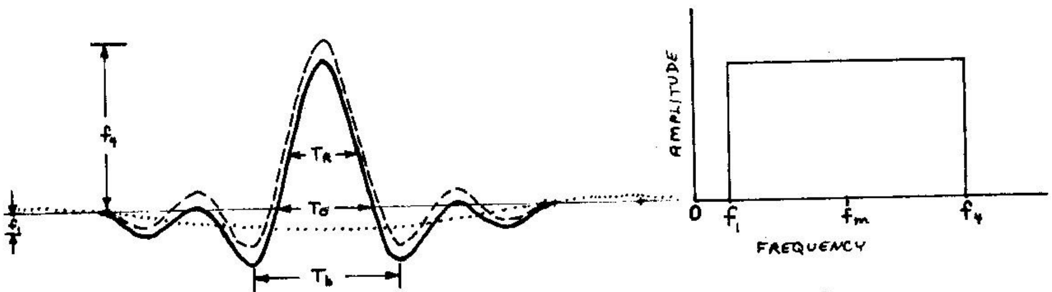
TEMPORAL RESOLUTION (T_R)

THE BAND-PASS SINC WAVELET

$$K_t = \frac{2 f_4 \sin(2\pi f_4 t)}{(2\pi f_4 t)} - \frac{2 f_1 \sin(2\pi f_1 t)}{(2\pi f_1 t)}$$



$$f_m = \frac{f_1 + f_4}{2}$$

TEMPORAL RESOLUTION $T_R = \frac{1}{1.5 f_4}$

WAVELET BREADTH $T_b = \frac{1}{0.7 f_4}$

PEAK-TO-TROUGH $T_b/2 = \frac{1}{1.4 f_4}$

$\left.\right\}$ $\frac{f_4}{f_1} \geqq 4$ (2 OCTAVES)

1st ZERO CROSSINGS $T_0 = \frac{1}{2 f_m}$ ALL OCTAVES

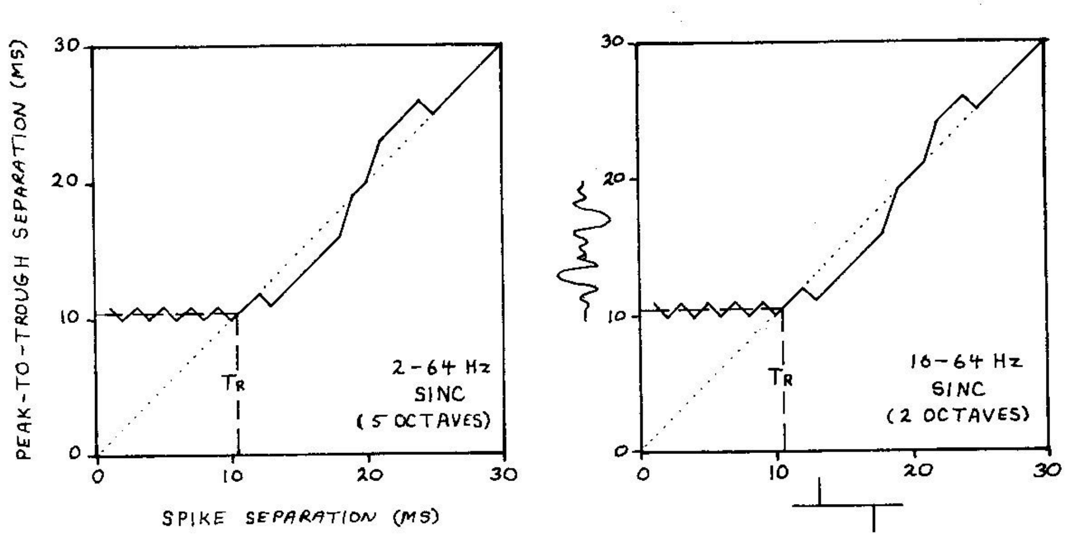
$T_R = 0.47\ T_b = 0.93\ T_b/2$ FOR SINCS $\geqq$ 2 OCTAVES

T_R OF SINC AND RICKER WAVELETS ARE EQUAL WHEN:

$$f_1 \text{ (PEAK FREQ. RICKER)} = \frac{f_4 \text{ (SINC)}}{2}$$

FIG. 6

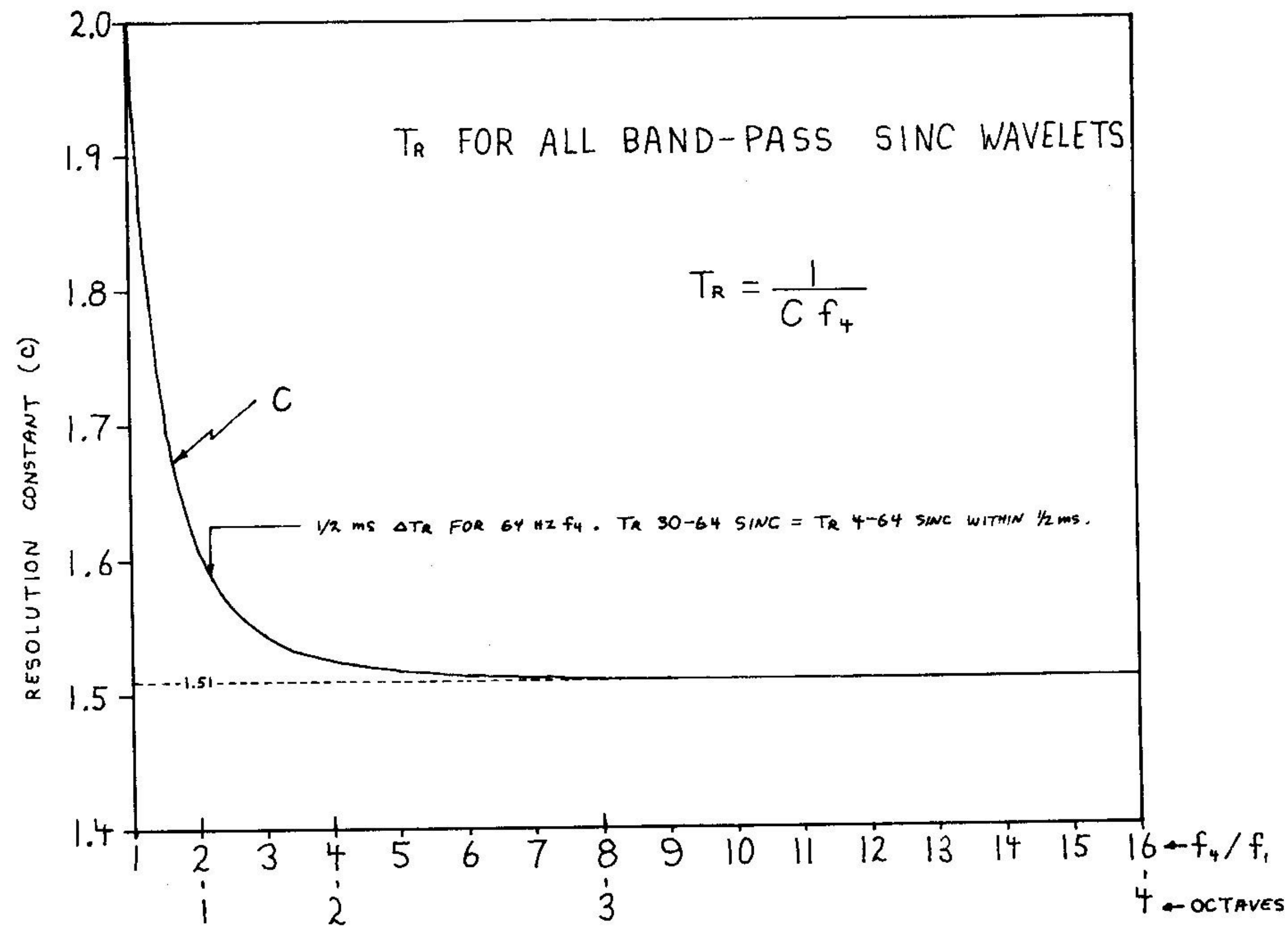
EFFECTS OF DECREASING BAND-WIDTH OF SINC WAVELET, WHILE HOLDING $f_{(max)}$ CONSTANT, ON TEMPORAL RESOLUTION



CONCLUSION: TEMPORAL RESOLUTION IS THE SAME FOR ALL SINC WAVELETS OF 2 OCTAVES AND GREATER BANDWIDTHS HAVING THE SAME $f_{(max)}$.

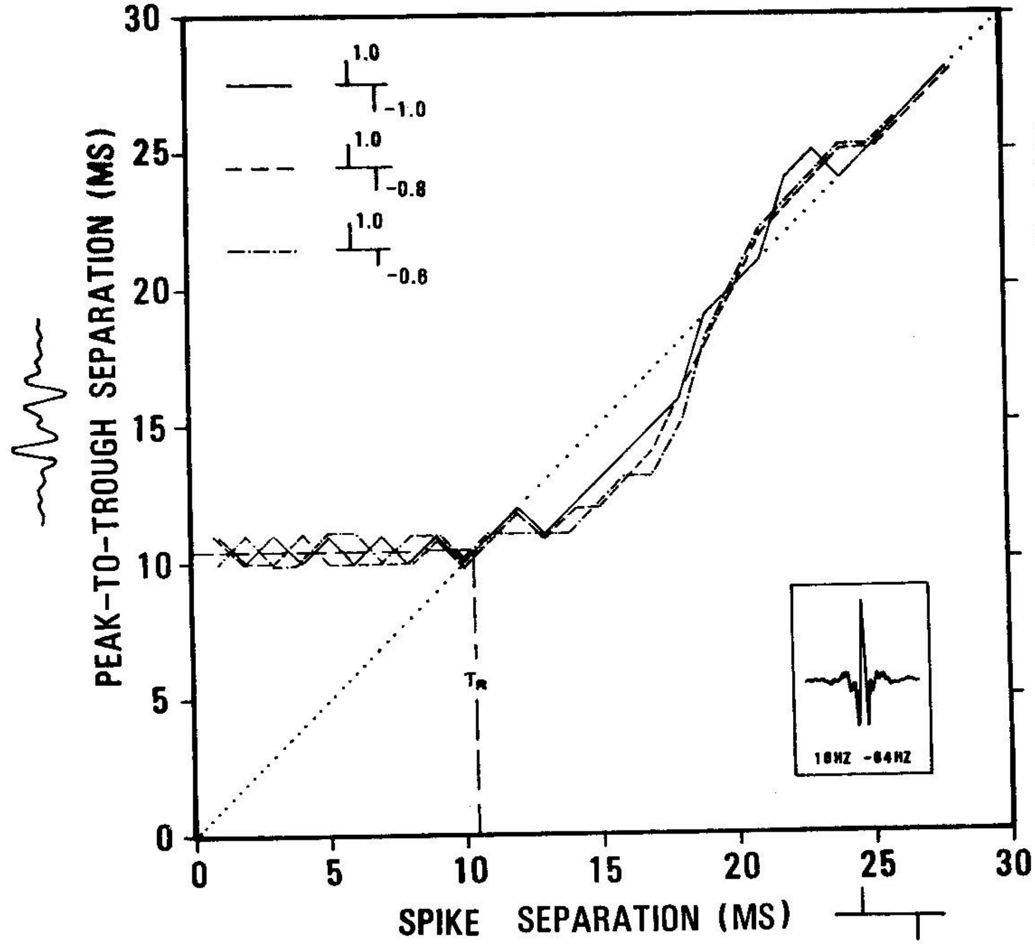
$$T_R = \frac{1}{1.5 f_{(max)}}$$

FIG. 7



T_R FOR ALL BAND-PASS SINC WAVELETS
T_R = 1 / C f_4
C
1/2 ms ΔT_R FOR 64 HZ f_4 . T_R 30-64 SINC = T_R 4-64 SINC WITHIN 1/2 ms.
1.51
RESOLUTION CONSTANT (C)
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 ← f_4/f_1
1 2 3 4 ← OCTAVES

FIG. 8



TEMPORAL RESOLUTION (T_R)

THE SINC WAVELET (16-64 HZ 2-OCTAVE) CONVOLVED WITH ALTERNATE POLARITY SPIKE PAIRS OF UNEQUAL AMPLITUDES.

FIG. 9